Largest Terrestrial Electromagnetic Pulsar

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The power spectrum formula of the synchrotron radiation generated by the electron and positron moving at the opposite angular velocities in homogenous magnetic field is derived in the Schwinger version of quantum field theory. It is surprising that the spectrum depends periodically on radiation frequency ω which means that the system composed from electron, positron, and magnetic field forms the pulsar.

KEY WORDS: source theory; spectral formula; synchrotron radiation; two-body system.

1. INTRODUCTION

We will show that the large hadron collider (LHC) which is at present time under construction in CERN can be considered in near future also as the largest electromagnetic terrestrial pulsar. We know it will operate with proton and antiproton beams in such a way that the collision center of mass energy will be 14 TeV and luminosity 10^{34} cm^{-1} s^{-2}. To achieve such large luminosity it must operate with more than 2800 bunches per beam and a very high density of particles in bunches. The LHC will also operate for heavy Pb ion physics at a luminosity of 10^{27} cm^{-1} s^{-2} (Evans, 1999).

The collision of particles is caused by the opposite directional motion of bunches. Or, if one bunch has the angular velocity ω, then the bunch with antiparticles has angular velocity $-\omega$. Here we will determine the spectral density of photons in the simplified case where one electron and one positron move in the opposite direction on a circle. We will show that the synergic spectrum depends periodically on time. This means that the behavior of the system is similar to the behavior of electromagnetic pulsar. In case the situation is generalized to bunches of LHC, then it is possible to call this accelerator a terrestrial electromagnetic pulsar. Now let us approach the theory and explicit calculation of the spectrum.

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The production of photons by circular motion of charged particle in accelerator is one of the most interesting problems in the classical and quantum electrodynamics.

In this paper we are interested in the synergic photon production initiated by the circular motion of electron and positron in the homogenous magnetic field. This process is the generalization of the one-charge synergic synchrotron Čerenkov radiation which has been calculated in source theory two decades ago by Schwinger *et al.* (1976). We will follow also the author's article (Pardy, 1994a) as the starting point. Although our final problem is the radiation of the two-charge system in vacuum, we consider, first in general, the presence of dielectric medium, which is represented by the phenomenological index of refraction n and it is well known that this phenomenological constant depends on the external magnetic field. Introducing the phenomenological constant enables to consider also the Čerenkov processes.

We will investigate here how the original Schwinger *et al.*'s spectral formula of the synergic synchrotron Čerenkov radiation of the charged particle is modified if we consider the electron and positron moving at opposite angular velocities. Later we put $n = 1$. This problem is an analogue of the linear and circular problem solved recently by author (Pardy, 1997, 2000) also in source theory. We will show that the original spectral formula of the synergic synchrotron-Čerenkov radiation is modulated by function $4 \sin^2(\omega t)$ where ω is the frequency of the synergic radiation produced by the system and it does not depend on the orbital angular frequency of electron or positron. We will use here the fundamental ingredients of Schwinger source theory to determine the power spectral formula.

Source theory (Dittrich, 1978; Schwinger, 1970, 1973) was initially constructed for a description of the particle physics situations occurring in high-energy physics experiments. It enables simplification of the calculations in the electrodynamics and gravity where the interactions are mediated by the photon or graviton, respectively. It simplifies particularly the calculations with radiative corrections (Dittrich, 1978; Pardy, 1994b).

2. FORMULATION OF A PROBLEM

The basic formula of the Schwinger source theory is the so-called vacuum to vacuum amplitude:

$$\langle 0_+ \mid 0_- \rangle = e^{\frac{i}{\hbar} W}, \tag{1}$$

where in case of the electromagnetic field in the medium, the action W is given by the following formula:

$$W = \frac{1}{2c^2} \int (dx)(dx') J^\mu(x) D_{+\mu\nu}(x - x') J^\nu(x'), \tag{2}$$

where

$$D_+^{\mu\nu} = \frac{\mu}{c}[g^{\mu\nu} + (1 - n^{-2})\beta^\mu\beta^\nu]D_+(x - x'), \tag{3}$$

where $\beta^\mu \equiv (1, \mathbf{0})$, $J^\mu \equiv (c\varrho, \mathbf{J})$ is the conserved current, μ is the magnetic permeability of the medium, ϵ is the dielectric constant of the medium, and $n = \sqrt{\epsilon\mu}$ is the index of refraction of the medium. Function D_+ is defined as follows (Schwinger *et al.*, 1976):

$$D_+(x - x') = \frac{i}{4\pi^2 c} \int_0^\infty d\omega \, \frac{\sin \frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t-t'|}. \tag{4}$$

The probability of the persistence of vacuum follows from the vacuum amplitude (1) in the following form:

$$|\langle 0_+ \mid 0_- \rangle|^2 = e^{-\frac{2}{\hbar}\text{Im}\,W}, \tag{5}$$

where Im W is the basis for the following definition of the spectral function $P(\omega, t)$:

$$-\frac{2}{\hbar}\,\text{Im}\,W \overset{d}{=} -\int dt\,d\omega\frac{P(\omega, t)}{\hbar\omega}. \tag{6}$$

Now, if we insert Eq. (2) into Eq. (6), we get after extracting $P(\omega, t)$ the following general expression for this spectral function:

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}\,d\mathbf{x}'\,dt'\left[\frac{\sin \frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|}\right]$$

$$\times \cos[\omega(t - t')]\left[\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}', t')\right]. \tag{7}$$

Let us recall that the last formula can be derived also in the classical electrodynamical context as it is shown for instance in the Schwinger article (Schwinger, 1949). The derivation of the power spectral formula from the vacuum amplitude is more simple.

3. THE POWER SPECTRUM OF OPPOSITELY MOVING CHARGES

Now, we will apply the formula (7) to the two-body system with the opposite charges moving at the opposite angular velocities in order to get in general synergic synchrotron-Čerenkov radiation of electron and positron moving in a uniform magnetic field.

While the synchrotron radiation is generated in a vacuum, the synergic synchrotron-Čerenkov radiation can produced only in a medium with dielectric constant n. We suppose the circular motion with velocity $\mathbf{v}$ in the plane perpendicular to the direction of the constant magnetic field $\mathbf{H}$ (chosen to be in the $+z$ direction).

The condition for the existence of the Čerenkov electromagnetic radiation is that the velocity of a charged particle in a medium is faster than the speed of light in this medium. This radiation was first observed experimentally by Čerenkov (1937) and theoretically interpreted by Tamm and Frank (1937) in the framework of classical electrodynamics. A source theoretical description of this effect was given by Schwinger *et al.* (1976) at the zero-temperature regime and the classical spectral formula was generalized to the finite temperature situation in electrodynamics and gravity in the framework of the source theory by author (Pardy, 1989, 1995). We derive the general formula for the synergic generation of radiation initiated by the motion of two-body system in a uniform magnetic field.

We can write the following formulas for the charge density ϱ and for the current density $\mathbf{J}$ of the two-body system with opposite charges and opposite angular velocities:

$$\varrho(\mathbf{x}, t) = e\delta(\mathbf{x} - \mathbf{x}_1(t)) - e\delta(\mathbf{x} - \mathbf{x}_2(t)) \tag{8}$$

and

$$\mathbf{J}(\mathbf{x}, t) = e\mathbf{v}_1(t)\delta(\mathbf{x} - \mathbf{x}_1(t)) - e\mathbf{v}_2(t)\delta(\mathbf{x} - \mathbf{x}_2(t)) \tag{9}$$

with

$$\mathbf{x}_1(t) = \mathbf{x}(t) = R(\mathbf{i}\,\cos(\omega_0 t) + \mathbf{j}\,\sin(\omega_0 t)), \tag{10}$$

$$\mathbf{x}_2(t) = R(\mathbf{i}\cos(-\omega_0 t) + \mathbf{j}\,\sin(-\omega_0 t)) = \mathbf{x}(-\omega_0, t) = \mathbf{x}(-t). \tag{11}$$

The absolute values of velocities of both particles are the same, or $|\mathbf{v}_1(t)| = |\mathbf{v}_2(t)| = v$, where ($H = |\mathbf{H}|$, $E =$ energy of a particle)

$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt}, \quad \omega_0 = \frac{v}{R}, \quad R = \frac{\beta E}{eH}, \quad \beta = \frac{v}{c}, \quad v = |\mathbf{v}|. \tag{12}$$

After insertion of Eqs. (8)–(9) into Eq. (7), and after some mathematical operations we get

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}e^2\int_{-\infty}^{\infty} dt'\,\cos(t - t')\sum_{i,j=1}^{2}(-1)^{i+j}$$

$$\times\left[1 - \frac{\mathbf{v}_i(t)\cdot\mathbf{v}_j(t')}{c^2}n^2\right]\left\{\frac{\sin\frac{n\omega}{c}|\mathbf{x}_i(t) - \mathbf{x}_j(t')|}{|\mathbf{x}_i(t) - \mathbf{x}_j(t')|}\right\}. \tag{13}$$

Using $t' = t + \tau$, we get for

$$\mathbf{x}_i(t) - \mathbf{x}_j(t') \overset{d}{=} \mathbf{A}_{ij}, \tag{14}$$

$$|\mathbf{A}_{ij}| = [R^2 + R^2 - 2RR\,\cos(\omega_0\tau + \alpha_{ij})]^{1/2} = 2R\left|\sin\left(\tfrac{\omega_0\tau + \alpha_{ij}}{2}\right)\right|, \tag{15}$$

where α_{ij} were evaluated as follows:

$$\alpha_{11} = 0, \quad \alpha_{12} = 2\omega_0 t, \quad \alpha_{21} = 2\omega_0 t, \quad \alpha_{22} = 0. \tag{16}$$

Using

$$\mathbf{v}_i(t) \cdot \mathbf{v}_j(t + \tau) = \omega_0^2 R^2 \cos(\omega_0 \tau + \alpha_{ij}), \tag{17}$$

and relation (15) we get with $v = \omega_0 R$

$$P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{\mu}{n^2} e^2 \int_{-\infty}^{\infty} d\tau \, \cos \omega \tau \sum_{i,j=1}^{2} (-1)^{i+j}$$

$$\times \left[1 - \frac{n^2}{c^2} v^2 \cos(\omega_0 \tau + \alpha_{ij}) \right] \left\{ \frac{\sin\left[\frac{2Rn\omega}{c} \sin\left(\frac{\omega_0 \tau + \alpha_{ij}}{2} \right) \right]}{2R \sin\left(\frac{\omega_0 \tau + \alpha_{ij}}{2} \right)} \right\}. \tag{18}$$

Introducing new variable T by relation

$$\omega_0 \tau + \alpha_{ij} = \omega_0 T \tag{19}$$

for every integral in Eq. (18), we get $P(\omega, t)$ in the following form:

$$P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{e^2}{2R} \frac{\mu}{n^2} \int_{-\infty}^{\infty} dT \sum_{i,j=1}^{2} (-1)^{i+j} \cos\left(\omega T - \frac{\omega}{\omega_0} \alpha_{ij} \right)$$

$$\times \left[1 - \frac{c^2}{n^2} v^2 \cos(\omega_0 T) \right] \left\{ \frac{\sin\left[\frac{2Rn\omega}{c} \sin\left(\frac{\omega_0 T}{2} \right) \right]}{\sin\left(\frac{\omega_0 T}{2} \right)} \right\}. \tag{20}$$

The last formula can be written in the more compact form,

$$P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{\mu}{n^2} \frac{e^2}{2R} \sum_{i,j=1}^{2} (-1)^{i+j} \left\{ P_1^{(ij)} - \frac{n^2}{c^2} v^2 P_2^{(ij)} \right\}, \tag{21}$$

where

$$P_1^{(ij)} = J_{1a}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij} + J_{1b}^{(ij)} \sin \frac{\omega}{\omega_0} \alpha_{ij} \tag{22}$$

and

$$P_2^{(ij)} = J_{2A}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij} + J_{2B}^{(ij)} \sin \frac{\omega}{\omega_0} \alpha_{ij}, \tag{23}$$

where

$$J_{1a}^{(ij)} = \int_{-\infty}^{\infty} dT \, \cos \omega T \left\{ \frac{\sin\left[\frac{2Rn\omega}{c} \sin\left(\frac{\omega_0 T}{2} \right) \right]}{\sin\left(\frac{\omega_0 T}{2} \right)} \right\}, \tag{24}$$

$$J_{1b}^{(ij)} = \int_{-\infty}^{\infty} dT \, \sin \omega T \left\{ \frac{\sin \left[\frac{2Rn\omega}{c} \sin \left(\frac{\omega_0 T}{2} \right) \right]}{\sin \left(\frac{\omega_0 T}{2} \right)} \right\}, \tag{25}$$

$$J_{2A}^{(ij)} = \int_{-\infty}^{\infty} dT \, \cos \omega_0 T \, \cos \omega T \left\{ \frac{\sin \left[\frac{2Rn\omega}{c} \sin \left(\frac{\omega_0 T}{2} \right) \right]}{\sin \left(\frac{\omega_0 T}{2} \right)} \right\}, \tag{26}$$

$$J_{2B}^{(ij)} = \int_{-\infty}^{\infty} dT \, \cos \omega_0 T \, \sin \omega T \left\{ \frac{\sin \left[\frac{2Rn\omega}{c} \sin \left(\frac{\omega_0 T}{2} \right) \right]}{\sin \left(\frac{\omega_0 T}{2} \right)} \right\}. \tag{27}$$

Using

$$\omega_0 T = \varphi + 2\pi l, \qquad \varphi \in (-\pi, \pi), \quad l = 0, \pm 1, \pm 2, \ldots, \tag{28}$$

we can transform the T-integral into the sum of the telescopic integrals according to the scheme:

$$\int_{-\infty}^{\infty} dT \rightarrow \frac{1}{\omega_0} \sum_{l=-\infty}^{l=\infty} \int_{-\pi}^{\pi} d\varphi. \tag{29}$$

Using the fact that for the odd functions $f(\varphi)$ and $g(l)$, the relations are valid

$$\int_{-\pi}^{\pi} f(\varphi) \, d\varphi = 0; \quad \sum_{l=-\infty}^{l=\infty} g(l) = 0, \tag{30}$$

we can write

$$J_{1a}^{(ij)} = \frac{1}{\omega_0} \sum_l \int_{-\pi}^{\pi} d\varphi \left\{ \cos \frac{\omega}{\omega_0} \varphi \, \cos 2\pi l \frac{\omega}{\omega_0} \right\} \left\{ \frac{\sin \left[\frac{2Rn\omega}{c} \sin \left(\frac{\varphi}{2} \right) \right]}{\sin \left(\frac{\varphi}{2} \right)} \right\}, \tag{31}$$

$$J_{1b}^{(ij)} = 0. \tag{32}$$

For integrals with indices A, B we get

$$J_{2A}^{(ij)} = \frac{1}{\omega_0} \sum_l \int_{-\pi}^{\pi} d\varphi \, \cos \varphi \left\{ \cos \frac{\omega}{\omega_0} \varphi \, \cos 2\pi l \frac{\omega}{\omega_0} \right\} \left\{ \frac{\sin \left[\frac{2Rn\omega}{c} \sin \left(\frac{\varphi}{2} \right) \right]}{\sin \left(\frac{\varphi}{2} \right)} \right\}, \tag{33}$$

$$J_{2B}^{(ij)} = 0. \tag{34}$$

So, the power spectral formula (21) is of the form

$$P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{\mu}{n^2} \frac{e^2}{2R} \sum_{i,j=1}^{2} (-1)^{i+j} \left\{ P_1^{(ij)} - n^2 \beta^2 P_2^{(ij)} \right\}; \quad \beta = \frac{v}{c}, \tag{35}$$

where

$$P_1^{(ij)} = J_{1a}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij},$$
(36)

$$P_2^{(ij)} = J_{2A}^{(ij)} \cos \frac{\omega}{\omega_0} \alpha_{ij}.$$
(37)

Using the Poisson theorem

$$\sum_{l=-\infty}^{\infty} \cos 2\pi \frac{\omega}{\omega_0} l = \sum_{k=-\infty}^{\infty} \omega_0 \delta(\omega - \omega_0 l),$$
(38)

the definition of the Bessel functions J_{2l} and their corresponding derivations and integrals

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \, \cos\left(z \, \sin \frac{\varphi}{2}\right) \cos l\varphi = J_{2l}(z),$$
(39)

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \, \sin\left(z \, \sin \frac{\varphi}{2}\right) \sin(\varphi/2) \cos l\varphi = -J_{2l}'(z),$$
(40)

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \, \frac{\sin\left(z \, \sin \frac{\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)} \cos l\varphi = \int_0^z J_{2l}(x) \, dx,$$
(41)

and using equations

$$\sum_{i,j=1}^{2} (-1)^{i+j} \cos \frac{\omega}{\omega_0} \alpha_{ij} = 2(1 - \cos 2\omega t) = 4 \sin^2 \omega t,$$
(42)

we get with the definition of the partial power spectrum P_l

$$P(\omega) = \sum_{l=1}^{\infty} \delta(\omega - l\omega_0) P_l,$$
(43)

the following final form of the partial power spectrum generated by motion of two-charge system moving in the cyclotron:

$$P_l(\omega, t) = [4(\sin \omega t)^2] \frac{e^2}{\pi n^2} \frac{\omega \mu \omega_0}{v}$$

$$\times \left(2n^2 \beta^2 J_{2l}'(2ln\beta) - (1 - n^2\beta^2) \int_0^{2ln\beta} dx \, J_{2l}(x)\right).$$
(44)

So we see that the spectrum generated by the system of electron and positron is formed in such a way that the original synchrotron spectrum generated by electron is modulated by function $4 \sin^2(\omega t)$. The derived formula involves also the synergic process composed from the synchrotron radiation and the Čerenkov radiation for electron velocity $v > c/n$ in a medium.

Our goal is to apply the last formula in situation where there is a vacuum. In this case we can put $\mu = 1$, $n = 1$ in the last formula and so we have

$$P_l(\omega, t) = 4 \, \sin^2(\omega t) \frac{e^2}{\pi} \frac{\omega \omega_0}{v} \left(2\beta^2 J'_{2l}(2l\beta) - (1 - \beta^2) \int_0^{2l\beta} dx \, J_{2l}(x) \right). \quad (45)$$

So, we see, that final formula describing the opposite motion of electron and positron in accelerator is of the form

$$P_{l,\text{pair}}(\omega, t) = 4 \, \sin^2(\omega t) P_{l(\text{electron})}(\omega), \quad (46)$$

where P_{electron} is the spectrum of radiation only of electron. The result is surprising because we naively expected that the total radiation of the opposite charges should be

$$P_l(\omega, t) = P_{l(\text{electron})}(\omega, t) + P_{l(\text{positron})}(\omega, t). \quad (47)$$

So, we see that the resulting radiation can not be considered as generated by the isolated particles but by a synergical production of a system of particles and magnetic field. At the same time we cannot interpret the result as a result of interference of two sources because the distance between sources radically changes and so, the condition of an interference is not fulfilled.

The classical electrodynamics formula (46) changes our naive opinion on the electrodynamical processes in the magnetic field. From the last formula also it follows that at time $t = \pi k/\omega$ there is no radiation of the frequency ω. The spectrum oscillates with frequency ω. If the radiation were generated not synergically, then the spectral formula would be composed from two parts corresponding to two isolated sources.

4. DISCUSSION

We have derived in this paper the power spectrum formula of the synchrotron radiation generated by the electron and positron moving at the opposite angular velocities in homogenous magnetic field. It forms an analogue of the author article (Pardy, 2000) where only colinear motion of two electrons, or two positrons was considered. We have used the Schwinger version of quantum field theory, for its simplicity. It is surprising that the spectrum depends periodically on radiation frequency ω which means that the system composed from electron, positron, and magnetic field behaves as a pulsating system. While such pulsar can be represented by a terrestrial experimental arrangement it is possible to consider also the cosmological existence in some modified conditions.

To our knowledge, our result is not involved in the classical monographies on the electromagnetic theory and at the same time it was not still studied by the accelerator experts investigating the synchrotron radiation of bunches. This effect was not described in textbooks on classical electromagnetic field and on the

synchrotron radiation. We hope that sooner or later this effect will be verified by the accelerator physicists.

The radiative corrections obviously influence the synergic spectrum of photons (Pardy, 1994a,b). However, the goal of this paper is restricted only to the simple processes.

The particle laboratory LEP in CERN used instead of single electron and positron the bunches with 10^{10} electrons or positrons in one bunch of volume $300\,\mu\mathrm{m} \times 40\,\mu\mathrm{m} \times 0.01$ m. So, in some approximation we can replace the charge of electron and positron by the charges Q and $-Q$ of both bunches in order to get the realistic intensity of photons. Nevertheless, the synergic character of the radiation of two bunches moving at the opposite direction in a magnetic field is conserved.

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